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ON THE PROBLEM OF THE THEORY OF ELASTICITY FOR THE RADIALLY INHOMOGENEOUS TRANSVERSAL ISOTROPIC CYLINDER WITH A FIXED LATERAL SURFACE

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Abstract. Using the method of asymptotic integration of the equations of the theory of elasticity, the axisymmetric problem of the theory of elasticity for a radially inhomogeneous transversally isotropic cylinder of small thickness is studied. Suppose that the elastic moduli are arbitrary continuous functions of the radius of the cylinder. It is assumed that the side part of the cylinder is fixed, and stresses are set at the ends of the cylinder, leaving the cylinder in equilibrium. Solutions have been determined having the nature of a boundary layer and are localized at the ends of the cylinder. The first terms of its asymptotic expansion coincide with the Saint-Venant edge effect in the theory of plates. The nature of the stress-strain state has been studied. It is shown that some boundary layer solutions may not be extinguished by propagating far from the ends.

Keywords: equilibrium equations, elastic moduli, boundary layer, iteration process, Saint-Venant edge effect.
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1 Introduction

In the theory of shells, the study of inhomogeneous shells occupies a special place. Despite the existence of a number of applied theories of inhomogeneous shells, the areas of their applicability have received limited investigation. The existence of various applied theories for inhomogeneous shells poses a problem for studying from the position of equations of the theory of elasticity.

The study of problems of elasticity theory for a cylinder has been the main focus of a number of studies (Tokovyy & Ma, 2019). An asymptotic theory has been developed for a transversally isotropic cylinder of small thickness (Mekhtiev, 2019). The Almansi-Michell problem for an inhomogeneous anisotropic cylinder was studied using a numerical-analytical method (Lin & Dong, 2006). The influence of material heterogeneity on the stress-strain state of the cylinder was studied (Ieşan & Quintanilla, 2007; Jiann, 2008). Based on the spline collocation method and the finite element method, the stress-strain state of a radially inhomogeneous cylinder was studied (Grigorenko & Yaremchenko, 2016; 2019). An analysis of a radially inhomogeneous cylinder subjected to uniform internal pressure is studied (Tutuncu & Temel, 2009). The thermomechanical behavior of hollow radially inhomogeneous cylinders has been studied (Jiann-Quo,

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2001; Liew et al., 2003). The behavior of the solution to a problem of elasticity theories for a radially inhomogeneous transversally isotropic cylinder has been studied using the asymptotic method (Akhmedov & Akperova, 2011). The purpose of this work is to study the behavior of solutions to the problem of elasticity theory for a transversally isotropic cylinder with a fixed lateral surface.

2 Statement of Boundary Value Problems for a Radially Inhomogeneous Transversal Isotropic Cylinder

Let us consider an axisymmetric elasticity problem for a radially inhomogeneous transversely isotropic cylinder of small thickness. Let's say a cylinder occupies a volume (Fig.1)

$$\Gamma = \{r \in [r_1; r_2], \varphi \in [0; 2\pi], z \in [-L; L]\}$$

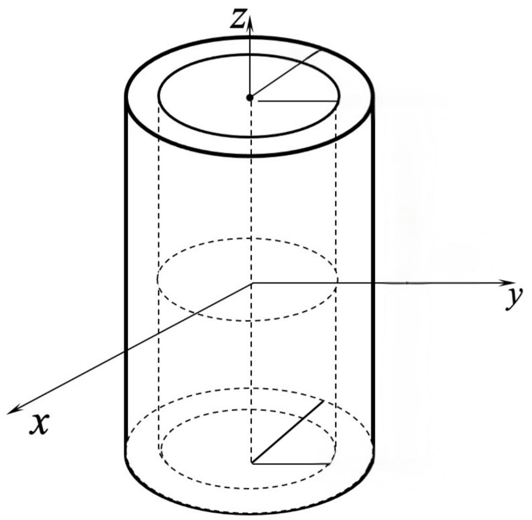

Figure 1: Radially inhomogeneous transversal isotropic cylinder

The system of equilibrium equations in the absence of mass forces in the cylindrical coordinate system r, φ, z has the form (Lur'e, 2005)

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0, \tag{1}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = 0. \tag{2}$$

Here $\sigma_{rr}, \sigma_{rz}, \sigma_{zz}, \sigma_{\varphi\varphi}$ - are components of the stress tensor, which are expressed through displacement vectors $u_r = u_r(r, z), u_z = u_z(r, z)$ as follows (Mekhtiev, 2019)

$$\sigma_{rr} = A_{11}\frac{\partial u_r}{\partial r} + A_{12}\frac{u_r}{r} + A_{13}\frac{\partial u_z}{\partial z}, \tag{3}$$

$$\sigma_{\varphi\varphi} = A_{12}\frac{\partial u_r}{\partial r} + A_{11}\frac{u_r}{r} + A_{13}\frac{\partial u_z}{\partial z}, \tag{4}$$

$$\sigma_{zz} = A_{13}(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}) + A_{33}\frac{\partial u_z}{\partial z}, \tag{5}$$

$$\sigma_{rz} = A_{44}(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}). \tag{6}$$

Here A_{ij} are the elasticity moduli.

Substituting (3)-(6) into (1), (2) the equation of equilibrium in displacements is obtained

$$\begin{gathered}\frac{\partial}{\partial \rho}\left[e^{-\varepsilon\rho}(b_{11}\frac{\partial u_\rho}{\partial \rho} + \varepsilon b_{12}u_\rho) + \varepsilon b_{13}\frac{\partial u_\xi}{\partial \xi}\right] + \\ + \varepsilon e^{-\varepsilon\rho}(b_{11} - b_{12})(\frac{\partial u_\rho}{\partial \rho} - \varepsilon u_\rho) + \varepsilon^2 e^{\varepsilon\rho} b_{44}\frac{\partial^2 u_\rho}{\partial \xi^2} + \varepsilon b_{44}\frac{\partial^2 u_\xi}{\partial \rho \partial \xi} = 0,\end{gathered} \tag{7}$$

$$\begin{gathered}\frac{\partial}{\partial \rho}\left[b_{44}(\varepsilon\frac{\partial u_\rho}{\partial \xi} + e^{-\varepsilon\rho}\frac{\partial u_\xi}{\partial \rho})\right] + \varepsilon b_{13}(\frac{\partial^2 u_\rho}{\partial \rho \partial \xi} + \varepsilon\frac{\partial u_\rho}{\partial \xi}) + \\ + \varepsilon^2 e^{\varepsilon\rho} b_{33}\frac{\partial^2 u_\xi}{\partial \xi^2} + \varepsilon b_{44}(\varepsilon\frac{\partial u_\rho}{\partial \xi} + e^{-\varepsilon\rho}\frac{\partial u_\xi}{\partial \rho}) = 0.\end{gathered} \tag{8}$$

Here $\rho = \frac{1}{\varepsilon}\ln\left(\frac{r}{r_0}\right), \xi = \frac{z}{r_0}$ - are new dimensionless variables; $\varepsilon = \frac{1}{2}\ln\left(\frac{r_2}{r_1}\right)$ - is a small parameter characterizing the thickness of the cylinder; $r_0 = \sqrt{r_1 r_2}, \rho \in [-1;1], \xi \in [-l;l], \left(l = \frac{L}{r_0}\right); u_\rho = \frac{u_r}{r_0}, u_\xi = \frac{u_z}{r_0}, b_{ij} = \frac{A_{ij}}{G_0}$ – are dimensionless quantities, G_0 - is some characteristic module, for example $G_0 = max|A_{ij}|$

Assume that the elastic moduli $b_{11} = b_{11}(\rho), b_{12} = b_{12}(\rho), b_{33} = b_{33}(\rho), b_{13} = b_{13}(\rho), b_{44} = b_{44}(\rho)$ is an arbitrary positive continuous functions of ρ a variable whose values can vary within the same order of magnitude.

Consider that the side surface of the cylinder is fixed, i.e.

$$u_\rho|_{\rho=\pm 1} = 0; u_\xi|_{\rho=\pm 1} = 0, \tag{9}$$

and the boundary conditions are specified at the ends of the cylinder

$$\sigma_{\rho\xi}|_{\xi=\pm l} = f_{1s}(\rho), \sigma_{\xi\xi}|_{\xi=\pm l} = f_{2s}(\rho). \tag{10}$$

Here $\sigma_{\rho\xi} = \frac{\sigma_{rz}}{G_0}, \sigma_{\xi\xi} = \frac{\sigma_{zz}}{G_0}$ - are dimensionless quantities; $f_{1s}(\rho), f_{2s}(\rho) (s = 1, 2)$ - are sufficiently smooth functions satisfying equilibrium conditions.

3 Construction of Solutions

Looking for solution (7), (8) in the form

$$u_\rho(\rho, \xi) = u(\rho)m'(\xi); u_\xi(\rho, \xi) = w(\rho)m(\xi), \tag{11}$$

where

$$m\prime\prime(\xi) - \mu^2 m(\xi) = 0 \tag{12}$$

and μ is the spectral parameter.

Substituting (11) into (7)-(9), and taking into account (12)

$$\begin{gathered}[e^{-\varepsilon\rho}(b_{11}u'(\rho) + \varepsilon b_{12}u(\rho)) + \varepsilon b_{13}w(\rho)]' + \varepsilon e^{-\varepsilon\rho}(b_{11} - b_{12})(u'(\rho) - \varepsilon u(\rho)) + \\ + \varepsilon b_{44}w'(\rho) + \mu^2\varepsilon^2 e^{\varepsilon\rho} b_{44}u(\rho) = 0,\end{gathered} \tag{13}$$

$$\begin{gathered}[b_{44}(e^{-\varepsilon\rho}w'(\rho) + \mu^2\varepsilon u(\rho))]' + \varepsilon b_{44}e^{-\varepsilon\rho}w'(\rho) + \\ + \varepsilon\mu^2 \times [b_{13}(u'(\rho) + \varepsilon u(\rho)) + \varepsilon b_{44}u(\rho) + \varepsilon e^{\varepsilon\rho} b_{33}w(\rho)] = 0,\end{gathered} \tag{14}$$

$$u(\rho)|_{\rho=\pm 1} = 0, \tag{15}$$

$$w(\rho)|_{\rho=\pm 1} = 0. \tag{16}$$

To solve (13)-(16), an asymptotic method based on three iterative processes (Goldenweiser, 1963; Akhmedov & Akperova, 2011; Akhmedov & Sofiyev, 2019) is used. The first iteration process corresponds to trivial homogeneous solutions. Solutions that have the nature of an edge effect, corresponding to the second asymptotic process for a cylinder with a fixed lateral surface, do not exist.

Based on the third iterative process, looking for solution (13)-(16) in the form

$$u(\rho) = \varepsilon^2 \beta_0^{-1}(u_0(\rho) + \varepsilon u_1(\rho) + ...), \tag{17}$$

$$w(\rho) = \varepsilon(w_0(\rho) + \varepsilon w_1(\rho) + ...), \tag{18}$$

$$\mu = \varepsilon^{-1}(\beta_0 + \varepsilon\beta_1 + ...). \tag{19}$$

Substituting (17)-(19) into (13)-(16), for the first terms of the expansion

$$[b_{11}u_0'(\rho) + b_{13}w_0(\rho)]' + b_{44}(w_0'(\rho) + \mu_0^2 u_0(\rho)) = 0, \tag{20}$$

$$[b_{44}(w_0'(\rho) + \mu_0^2 u_0(\rho))]' + \mu_0^2(b_{13}u_0'(\rho) + b_{33}w_0(\rho)) = 0, \tag{21}$$

$$u_0(\rho)|_{\rho=\pm 1} = 0, \tag{22}$$

$$w_0(\rho)|_{\rho=\pm 1} = 0. \tag{23}$$

Spectral problem (20)-(23) describes the potential solution of a transversely isotropic plate that is non-uniform in thickness. In contrast to the isotropic case, for a transversely isotropic plate of non-uniform thickness, β_{0k} can also take on purely imaginary values (Ustinov, 2006).

By the substitution:

$$u_0 = -\beta_0^{-3}(p_0\psi'')' + \beta_0^{-1}p_2\psi' + \beta_0^{-1}(p_1\psi)', \tag{24}$$

$$w_0 = \beta_0^{-2}p_0\psi'' - p_1\psi, \tag{25}$$

spectral problem (20)-(23) reduces to the following

$$(p_0\psi'')'' - \beta_0^2[(p_1\psi)'' + p_1\psi'' + (p_2\psi')'] + \beta_0^4 p_3\psi = 0, \tag{26}$$

$$(-\beta_0^{-3}(p_0\psi'')' + \beta_0^{-1}p_2\psi' + \beta_0^{-1}(p_1\psi)')|_{\rho=\pm 1} = 0, \tag{27}$$

$$(\beta_0^{-2}p_0\psi'' - p_1\psi)|_{\rho=\pm 1} = 0, \tag{28}$$

where $p_0 = b_{11}\theta, p_1 = b_{13}\theta, p_2 = b_{44}^{-1}, p_3 = b_{33}\theta, \theta = (b_{13}^2 - b_{11}b_{33})^{-1}$.

Problems (26)-(28) are the generalization of the spectral problem of P.F. Papkovich to the inhomogeneous transversely isotropic case (Akhmedov & Sofiyev, 2019).

The solutions corresponding to the third iterative process have the form

$$u_\rho(\rho;\varepsilon)=\varepsilon^2\sum_{k=1}^{\infty}\left[-\beta_{0k}^{-4}(p_0\psi_k'')'+\beta_{0k}^{-2}(p_2\psi_k'+(p_1\psi_k)')+O(\varepsilon)\right]m_k'(\xi), \tag{29}$$

$$u_\xi(\rho;\xi)=\varepsilon\sum_{k=1}^{\infty}\left[\beta_{0k}^{-2}p_0\psi_k''-p_1\psi_k+O(\varepsilon)\right]m_k(\xi). \tag{30}$$

Here $m_k(\xi)=C_k ch(\mu_k\xi)+D_k sh(\mu\xi), C_k$ and D_k are arbitrary constants.

For stresses the following asymptotic expressions are obtained

$$\sigma_{\rho\rho}=\varepsilon\sum_{k=1}^{\infty}(-\psi_k+O(\varepsilon))m_k'(\xi), \tag{31}$$

$$\sigma_{\rho\xi}=\sum_{k=1}^{\infty}(\psi_k'+O(\varepsilon))m_k(\xi), \tag{32}$$

$$\sigma_{\xi\xi}=\varepsilon\sum_{k=1}^{\infty}(-\beta_{0k}^{-2}\psi_k''+O(\varepsilon))m_k'(\xi), \tag{33}$$

$$\sigma_{\varphi\varphi}^{*}=\varepsilon\sum_{k=1}^{\infty}[(b_{13}p_0-b_{12}p_1)\beta_{0k}^{-2}\psi_k''+(b_{12}p_3-b_{13}p_1)\psi_k+O(\varepsilon)]m_k'(\xi). \tag{34}$$

Here $\sigma_{\rho\rho}=\frac{\sigma_{rr}}{G_0},\sigma_{\varphi\varphi}^{*}=\frac{\sigma_{\varphi\varphi}}{G_0}$ - are dimensionless quantities.

The third asymptotic process is determined by solutions (29), (30), which have the nature of a boundary layer. The first terms (29), (30) are equivalent to the Saint-Venant edge effect of an inhomogeneous transversely isotropic plate. For purely imaginary β_{0k}, the Saint-Venant boundary layer damps very weakly and solutions (29), (30) should be classified as penetrating solutions. Therefore, in this case, the stress-strain state of the transversally isotropic and isotropic cylinders is qualitatively different. When β_{0k} is not purely imaginary, the general picture of the stress-strain state is qualitatively similar to the corresponding picture for isotropic radially inhomogeneous cylinders. Quantitatively, they differ in the rate of attenuation of the Saint-Venant boundary layers.

To determine the constants C_k and D_k, Lagrange's variational principle (Lur'e, 2005; Mekhtiev, 2019) is used. The variational principle takes the following form

$$\sum_{s=1}^{2}\int_{-1}^{1}[(\sigma_{\rho\xi}-f_{1s})\delta u_\rho+(\sigma_{\xi\xi}-f_{2s})\delta u_\xi]|_{\xi=\pm l}e^{2\varepsilon\rho}d\rho=0. \tag{35}$$

Substituting (29), (30), (32), (33) into (35) and considering $\delta C_k,\delta D_k$ to be independent variations, to determine C_k and D_k an infinite system of equations is obtained

$$\sum_{k=1}^{\infty}F_{jk}^{(1)}C_{k0}=\tau_{1j}, \tag{36}$$

$$\begin{aligned}F_{jk}^{(1)}=&\beta_{0k}^{-1}sh(\frac{\beta_{0k}l}{\varepsilon})sh(\frac{\beta_{0j}l}{\varepsilon})\int_{-1}^{1}\psi_k''[-\beta_{0j}^{-2}p_0\psi_j''+p_1\psi_j]d\rho+\\&+\beta_{0j}^{-1}ch(\frac{\beta_{0j}l}{\varepsilon})ch(\frac{\beta_{0k}l}{\varepsilon})\int_{-1}^{1}\psi_k[p_1\psi_j''-\beta_{0j}^{2}p_3\psi_j]d\rho,\end{aligned}$$

$$\tau_{1j} = \frac{1}{2}\beta_{0j} ch(\frac{\beta_{0j}l}{\varepsilon}) \int_{-1}^{1} [-\beta_{0j}^{-4}(p_0\psi_j'')' + \beta_{0j}^{-2}(p_2\psi_j' + (p_1\psi_j)')](f_{11}(\rho) + f_{12}(\rho))d\rho +$$
$$+ \frac{1}{2} sh(\frac{\beta_{0j}l}{\varepsilon}) \int_{-1}^{1} (f_{22}(\rho) - f_{21}(\rho))(\beta_{0j}^{-2}p_0\psi_j'' - p_1\psi_j)d\rho,$$

$$\sum_{k=1}^{\infty} F_{jk}^{(2)} D_{k0} = \tau_{2j}, \qquad (37)$$

$$F_{jk}^{(2)} = \beta_{0k}^{-1} ch(\frac{\beta_{0k}l}{\varepsilon}) ch(\frac{\beta_{0j}l}{\varepsilon}) \int_{-1}^{1} \psi_k''[-\beta_{0j}^{-2}p_0\psi_j'' + p_1\psi_j]d\rho +$$
$$+ \beta_{0j}^{-1} sh(\frac{\beta_{0j}l}{\varepsilon}) sh(\frac{\beta_{0k}l}{\varepsilon}) \int_{-1}^{1} \psi_k[p_1\psi_j'' - \beta_{0j}^{2}p_3\psi_j]d\rho,$$

$$\tau_{2j} = \frac{1}{2}\beta_{0j} sh(\frac{\beta_{0j}l}{\varepsilon}) \int_{-1}^{1} [-\beta_{0j}^{-4}(p_0\psi_j'')' + \beta_{0j}^{-2}(p_2\psi_j' + (p_1\psi_j)')](f_{12}(\rho) - f_{11}(\rho))d\rho +$$
$$+ \frac{1}{2} ch(\frac{\beta_{0j}l}{\varepsilon}) \int_{-1}^{1} (\beta_{0j}^{-2}p_0\psi_j'' - p_1\psi_j)(f_{21}(\rho) + f_{22}(\rho))d\rho,$$

$$C_k = C_{k0} + \varepsilon C_{k1} + ..., D_k = D_{k0} + \varepsilon D_{k1} + ...$$

The definition of constants $C_{kp}, D_{kp} (p = 1, 2, ...)$ invariably comes down to systems whose matrices coincide with the matrices of systems (36), (37).

For (36), (37) solvability is studied (Ustinov, 2006).

4 Numerical analysis

As an example, consider the problem of the stress-strain state of a radially inhomogeneous and homogeneous cylinder of small thickness.

Assume that the side part of the cylinder is fixed, and the boundary conditions are specified at the ends of the cylinder

$$\sigma_{rz} = Ar, \sigma_{zz} = A(2r^2 + 3r), z = -1.5$$

$$\sigma_{rz} = Ar^2, \sigma_{zz} = A(r^2 + 4r), z = 1.5$$

For a radially inhomogeneous cylinder the following cases are considered

a) elastic moduli change linearly along the radius (increasing elastic moduli): $G = G_0 r$, $\lambda = \lambda_0 r$;

b) elastic moduli change inversely proportional to the laws along the radius (declining elastic moduli): $G = \frac{G_0}{r}, \lambda = \frac{\lambda_0}{r}$.

Let's study the following two cases:

1. Area occupied by the cylinder $\Gamma = \{r \in [1; 1.5], \varphi \in [0; 2\pi], z \in [-1.5; 1.5]\}$

The parameter characterizing the thickness of the cylinder is equal to $\varepsilon = 0,2$.

Figures 2-4 show the distributions along the thickness (along the center line) of displacement u_r, u_z, stress σ_{rz} for a homogeneous and radially inhomogeneous cylinder.

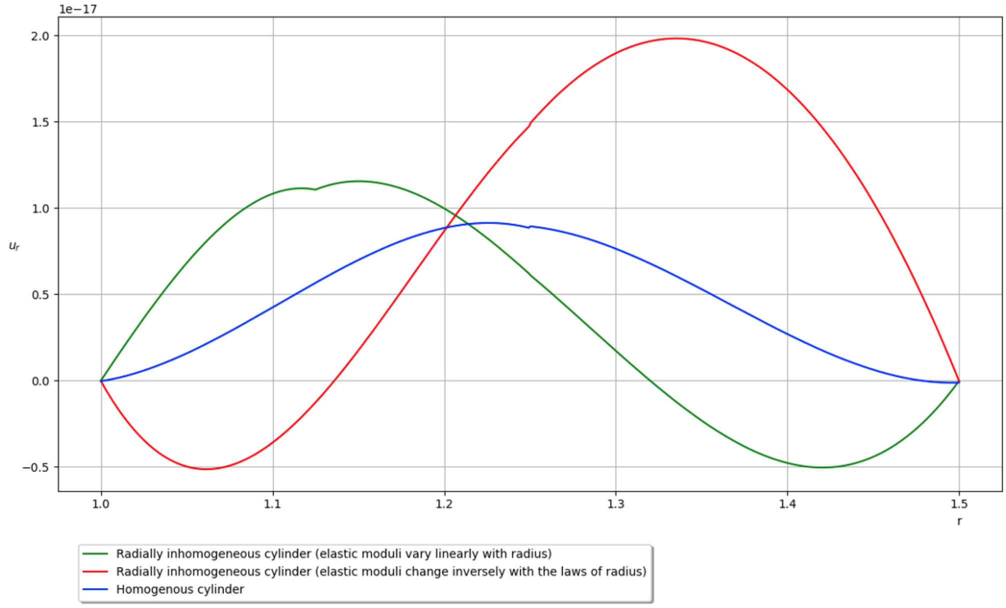

Figure 2: Distributions of u_r along the center line for $\varepsilon = 0.2$

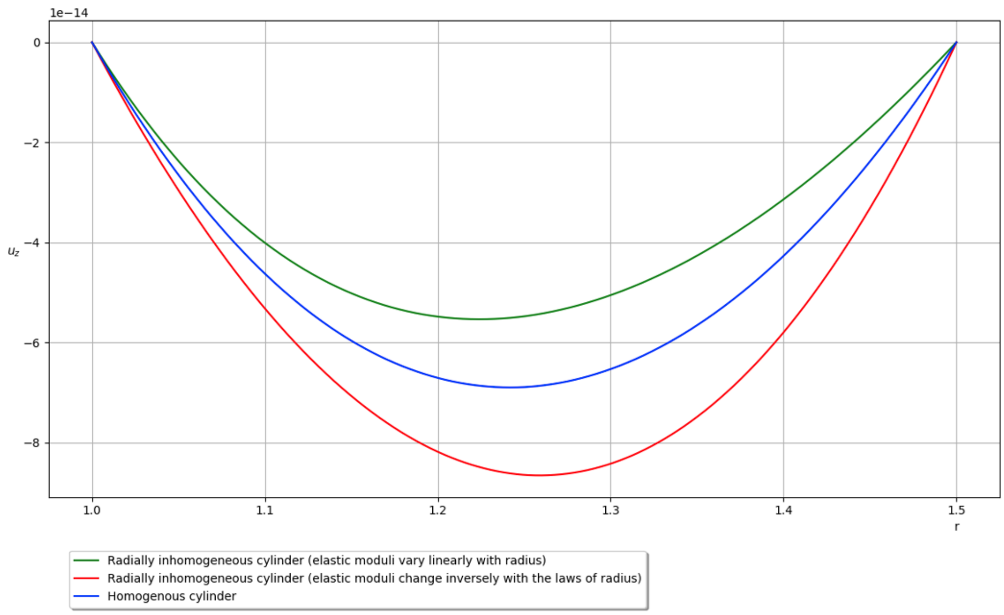

Figure 3: Distributions of u_z along the center line for $\varepsilon = 0.2$

For an inhomogeneous transverse-isotropic cylinder whose elasticity coefficients vary linearly with respect to the radius, the distribution of u_r along the thickness takes its largest value at a distance of 1.15 from the inner surface, and its smallest value at a distance of 0.0794 from the outer surface. For a non-homogeneous transverse-isotropic cylinder whose elasticity coefficients vary inversely proportional to the radius, the distribution of u_r along the thickness takes its smallest value at a distance of 1.0615 from the inner surface, and its largest value at a distance of 0.1646 from the outer surface. For a homogeneous transverse-isotropic cylinder at a distance of 1.2502 from the inner surface The distribution of u_r along the thickness increases until r=1.2502, and decreases from that point to the outer surface. The distribution of u_r along the thickness is qualitatively different for homogeneous and non-homogeneous transverse-isotropic cylinders (Fig. 2).

The distribution of u_z along the thickness qualitatively follows the same law (quadratic law), and the convexity of the parabolas describing those distributions is directed downwards (Fig. 3).

For homogeneous and non-homogeneous cylinders, the distribution of σ_{rz} along the thickness occurs according to the same law, and they differ from each other only quantitatively (Fig. 4).

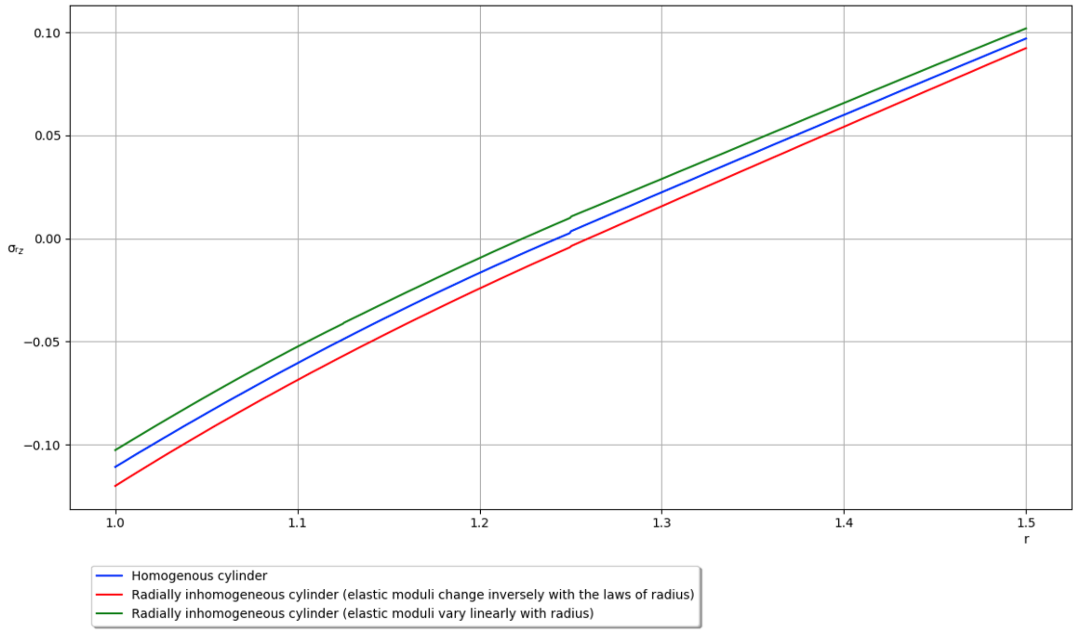

Figure 4: Distributions of σ_{rz} along the center line for $\varepsilon = 0.2$

2. Area occupied by a cylinder $\Gamma = \{r \in [1; 1.05], \varphi \in [0; 2\pi], z \in [-1.5; 1.5]\}$.

The parameter characterizing the thickness of the cylinder is $\varepsilon = 0.02$.

Figures 5-7 show the distribution of thickness u_r, u_z, σ_{rz} for homogeneous and radially inhomogeneous cylinders at $\varepsilon = 0.02$.

For an inhomogeneous transverse-isotropic cylinder whose elasticity coefficients vary linearly with respect to the radius, the distribution of u_r along the thickness takes its largest value at a distance of 1.025 from the inner surface, and its smallest value at a distance of 1.009 from the inner surface and 0.009 from the outer surface. For a uniform cylinder, the thickness distribution of u_r takes its largest value at a distance of 1.025 from the inner surface, and its smallest value at distances of 1.0091 from the inner surface and 1.04093 from the outer surface. For non-homogeneous cylinders, whose coefficients of elasticity vary linearly with respect to the radius, the distribution of u_r along the thickness qualitatively follows the same law. When the elasticity coefficients change inversely proportional to the radius, the distribution of u_r along the thickness takes rather small values (Fig. 5).

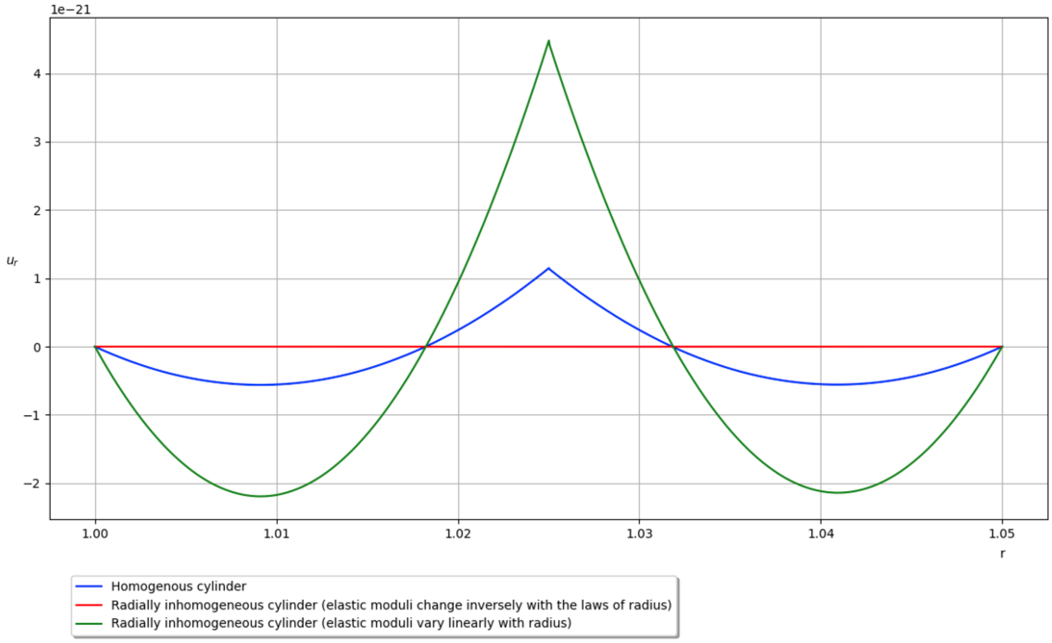

Figure 5: Distributions of u_r along the center line for $\varepsilon = 0.02$

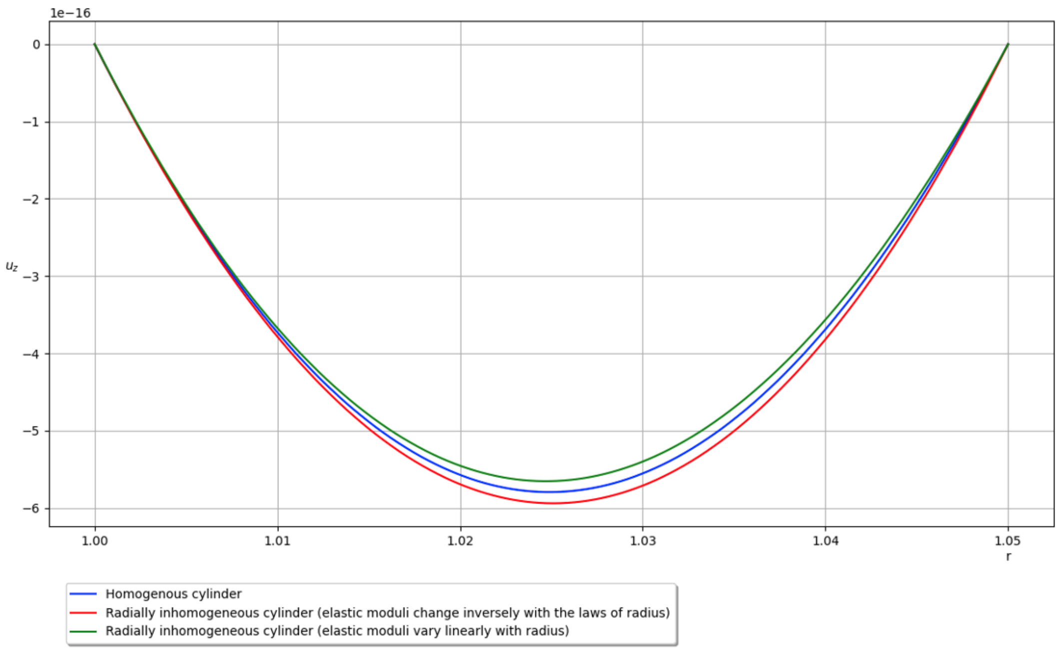

Figure 6: Distributions of u_z along the center line for $\varepsilon = 0.02$

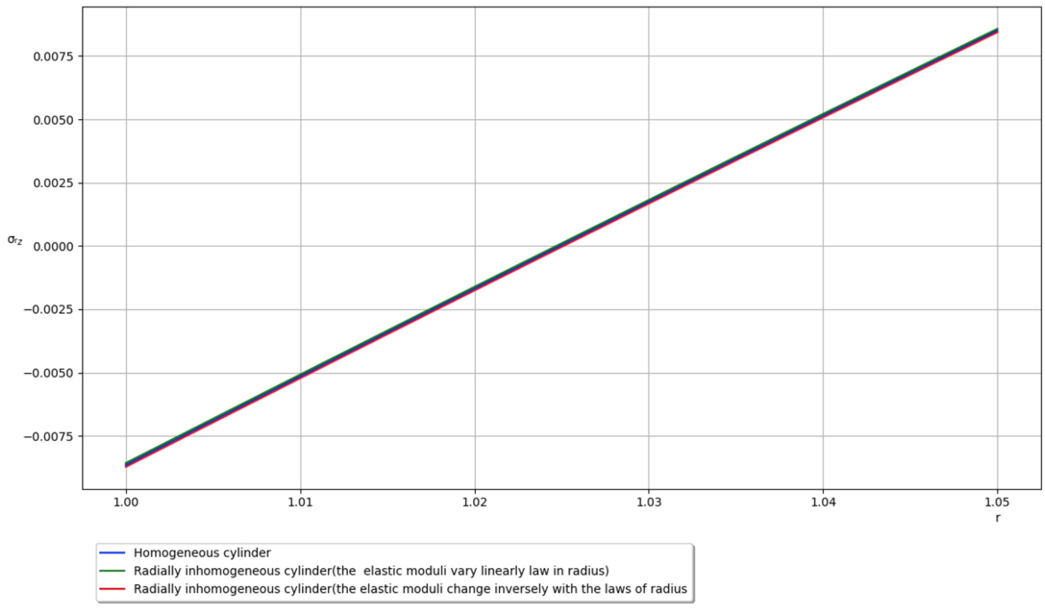

Figure 7: Distributions of σ_{rz} over the thickness of the cylinder in the case of $\varepsilon = 0.02$

The distribution of u_z along the thickness occurs according to the quadratic law for homogeneous and non-homogeneous cylinders. Those distributions differ only quantitatively (Fig. 6).

The distribution of σ_{rz} along the thickness occurs with a linear law for homogeneous and non-homogeneous cylinders and they almost coincide (Fig. 7).

From the analysis of numerical results it follows that the heterogeneity of the material can have a significant impact on the stress-strain state of the cylinder.

5 Conclusion

Based on an asymptotic analysis, it was found that when the lateral surface of a radially inhomogeneous transversally isotropic cylinder is fixed, the stress-strain state is composed only of a solution that has the nature of a boundary layer. Some boundary layer solutions attenuate very weakly and they can penetrate quite deeply away from the ends of the cylinder. Asymptotic formulas are obtained to calculate the stress-strain state of a radially inhomogeneous cylinder. As a result of the numerical calculation, the effect of the inhomogeneity of the material on the stress strain state of the transversally isotropic cylinder with small thickness is studied. It is determined that the assumption of the homogeneity of the material does not take into account some of its mechanical properties, and the inhomogeneity of the material is one of the main properties affecting the stress-strain state of elastic bodies.

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